THE SHEARING MODES APPROACH TO THE THEORY OF PLASMA SHEAR FLOWS

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First experimental evidences of the L-H transition and formation of the density transport barrier in ASDEX tokamak. (1982)

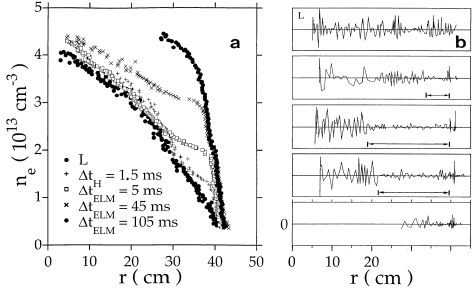


Figure 1. Evolution of density profiles and fluctuations following the L–H transition in ASDEX
Wagner F et al. 1991 Plasma Physics and Controlled Nuclear Fusion Research (Proc. 13th Int. Conf.,Washington, 1990) vol 1 (Vienna: IAEA) p 277;

(a) development of edge density profiles (from reflectometry) from the L-mode phase, 0.5 ms after the transition and then in a quiescent phase which developed after an ELMy phase;

(b) radial variation of edge fluctuations for the same time points.

First experimental evidences of the L-H transition and formation of the density transport barrier in Doublett-III-D tokamak. (1982.)

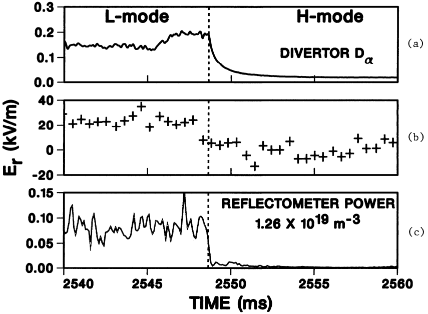


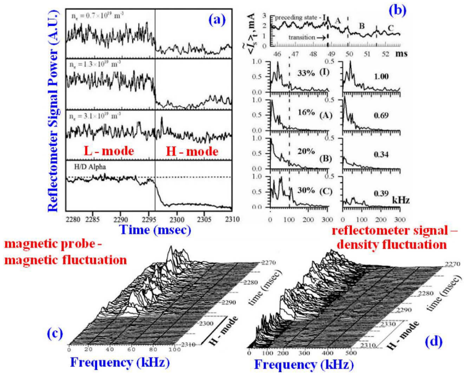
Figure. 2. Temporal changes at the L–H transition in DIII-D (Doyle E J *et al* 1993 *Plasma Physics and Controlled Nuclear Fusion Research (Proc. 14th Int. Conf.,W¨urzburg, 1992)* vol 1 (Vienna: IAEA) p 235:

(a) the drop in D*α*, signifying the transition;

(b) the change in *Er* ;

(c) the associated drop in density fluctuations as measured by a reflectometer.

4.pdf


(a)
Reflectometer Signal Power (A.U.)
L - mode
H - mode
H/D Alpha
Time (msec)
(b)
preceding state - I
A
B
C
transition
ms
33% (I)
16% (A)
20% (B)
30% (C)
1.00
0.69
0.34
0.39
kHz
magnetic probe -
magnetic fluctuation
(c)
Frequency (kHz)
time (msec)
H - mode
reflectometer signal –
density fluctuation
(d)
Frequency (kHz)
time (msec)
H - mode

The key aspects of the experimental observations on H-mode phenomena - regime of the enhanced confinement:

(1) The H-mode transition is an edge phenomena.

(2) The time scales associated with the H-mode transition usually comprise a fast sub-millisecond or microsecond onset, followed by a slower evolution on a time scale of 10 ms.

The temporal sequence of events occurring at the transition:

1. **The creation of steep radial electric field gradients**.
2. **Rapid suppression or reduction of the drift turbulence and anomalous transport.**
3. **Steepening of density and temperature profiles at the plasma edge.**
4. **Creation of an edge transport barrier, typically several centimetres wide.**
5. **Generation of edge localized modes (ELMs).**

Numerical simulations of the tokamak plasma stability in sheared flows

The first nonlinear gyro-Landau fluid simulations

R. E. Waltz, G. D. Kerbel, J. Milovich, Phys. Plasmas **1**, 2229 (1994),
R. E. Waltz, G.D. Kerbel, J. Milovich, G. W. Hammett, Phys. Plasmas **2**, 2404 (1995),
R. E. Waltz, R. L. Dewar, X. Garbet, Phys. Plasmas **5**, 1784 (1998),

reveal basic general qualitative result:

The turbulence is suppressed by shear flow, when the flow velocity shearing rate becomes larger than the maximum growth rate of the instabilities which can be developed, i.e.

$$\mathbf{V'_{0y} \equiv \frac{dV_{0y}}{dx} > \gamma_{max}}.$$

Beginning from experiments on Doublet–III–D tokamak this empiric quench rule, was confirmed experimentally in numerous experiments in tokamaks.

Shearing modes

The modal type perturbation in the frame of references convected with plasma flow

$$\phi(\mathbf{R}, t) = \phi_0 \exp(i\omega t - ik_x\xi - ik_y\eta)$$

with shearing velocity

$$V_0(\xi)\,\mathbf{e}_\eta = V_0'\xi\mathbf{e}_y.$$

In the laboratory set of reference

$$x = \xi, \quad y = \eta + V_0'\xi t \quad or \quad \xi = x, \quad \eta = y - V_0'xt$$

the perturbation has a non-modal structure:

$$\begin{aligned}\phi(\mathbf{r}, t) &= \phi_0 \exp\left(i\omega t - ik_x x - ik_y\left(y - V_0'xt\right)\right)\\ &= \phi_0 \exp\left(i\left(\omega + k_y V_0'x\right)t - ik_x x - ik_y y\right)\\ &= \phi_0 \exp\left(i\omega t - i\left(k_x - k_y\mathbf{V_0'}\mathbf{t}\right)x - ik_y y\right).\end{aligned}$$

For $k_x \sim k_y$, under the condition of the "quench rule" $\mathbf{V_0'} > \gamma$ and $t \sim \gamma^{-1}$, $\mathbf{V_0'}\mathbf{t} > 1$!!! **The non-modal term becomes dominant!**
Note, that in the normal-mode approach, the solution is sought in the normal mode (modal) form $\phi(\mathbf{r}, t) = \phi(x)\exp(ik_y y - i\omega t)$ with SEPARABLE dependences on time and x-coordinate.

Non-modal approach to the theory of the plasma shear flows stability

- The non–modal approach to the theory of the stability of the shear flows grounds on the methodology of the shearing modes.

- The shearing modes are the waves that have a static spatial structure in the frame of the background flow.

- They shear with the background flow.

Non–modal approach to sheared toroidal flows with applications to the ballooning instabilities:

W. A. Cooper. Ballooning instabilities in tokamaks with sheared toroidal flows, Plasma Physics and Controlled Fusion, Vol. 30, 1805-1812 (1988)

F. L. Waelbroeck, L. Chen, Ballooning instabilities in tokamaks with sheared toroidal flows, Phys.Fluids B 3, 601 (1990)

Non–modal approach to the theory of the magnetorotational instability:

J. Squire, A. Bhattacharjee, Nonmodal growth of the magnetorotational instability, Phys.Rev.Lett, 113, 025006 (2014)

J. Squire, A. Bhattacharjee, Magnetorotational instability: nonmodal growth and the relationship of global modes to the shearing box, The Astrophysical Journal, 797:67 (2014)

Non–modal approach to the theory of the diocotron instability:

V.V. Mikhailenko, Hae June Lee, V.S. Mikhailenko, Non-modal analysis of the diocotron instability: Plane geometry, Phys. Plasmas 19, 082112 (2012);

V.V. Mikhailenko, Hae June Lee, V.S. Mikhailenko, N.A.Azarenkov, Non-modal analysis of the diocotron instability: Cylindrical geometry, Phys. Plasmas 20, 042101 (2013)

V.V. Mikhailenko, V.S. Mikhailenko, Younghyun Jo, and Hae June Lee, Nonlinear shearing modes approach to the diocotron instability of a planar electron strip, Physics of Plasmas 22, 092125 (2015);

The application of the shearing modes approach to the fluid models of the plasma shear flows.

Hasegawa–Wakatani system of equations for plasma with strong shear flow

Mikhailenko V.S., Mikhailenko V.V., and Stepanov K.N., Temporal evolution of linear drift waves in a collisional plasma with homogeneous shear flow. Physics of Plasmas, 2000, vol.7, N.1, P.94–100.

We investigate the temporal evolution of drift modes in time-dependent shear flow using the Hasegawa–Wakatani system of equations for the dimensionless density $n = \tilde{n}/n_e$ and potential $\phi = e\varphi/T_e$ perturbations (n_e is the electron background density, T_e is the electron temperature)

$$\rho_s^2\left(\frac{\partial}{\partial t} + V_0\left(x,t\right)\frac{\partial}{\partial y} - \frac{cT_e}{eB}\left(\frac{\partial\phi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}\right)\right)\nabla^2\phi = a\frac{\partial^2}{\partial z^2}\left(n-\phi\right)$$

$$\left(\frac{\partial}{\partial t} + V_0\left(x,t\right)\frac{\partial}{\partial y} - \frac{cT_e}{eB}\left(\frac{\partial\phi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}\right)\right)n + v_{de}\frac{\partial\phi}{\partial y} = a\frac{\partial^2}{\partial z^2}\left(n-\phi\right)$$

Linear Hasegawa-Mima equation for $a \to \infty$ (adiabatic electrons):

$$\left(\frac{\partial}{\partial t} + V_0\left(x,t\right)\frac{\partial}{\partial y}\right)\left(\phi - \rho_s^2\nabla^2\phi\right) - v_{de}\frac{\partial\phi}{\partial y} = 0$$

In the normal-mode approach, the solution is sought in the form

$$\phi\left(\mathbf{r},t\right)=\phi\left(x\right)\exp\left(ik_y y-i\omega t\right)$$

and for the modal structure $\phi\left(x\right)$ we obtain the equation

$$\rho_s^2\frac{d^2\phi\left(x\right)}{dx^2}-\left[\left(1+k_y^2\rho_s^2\right)-\frac{k_y v_{de}}{\omega-k_y V_0\left(x\right)}\right]\phi\left(x\right)=0.$$

With new spatial variables ξ,η,

$$t=t,\qquad \xi=x,\qquad \eta=y-V_0'xt,\qquad z=z.$$

the Fourier transformed Hasegawa–Wakatani system reduces to the equation

$$\frac{1}{C}\frac{d^2}{dT^2}\left[\left(1+T^2\right)\phi\right]+\frac{d}{dT}\left\{\left[1+l^2\rho_s^2\left(1+T^2\right)\right]\phi\right\}+iSl\rho_s\phi=0,$$

where a dimensionless time variable T is defined by $T=V_0'\tau-\left(k_\perp/l\right)$ and parameters C and S are equal respectively to $C=\dfrac{ak_z^2}{\rho_s^2l^2V_0'}=\dfrac{T_ek_z^2}{\rho_s^2l^2\,V_0'n_0e^2\eta_\parallel}$, $S=\dfrac{lv_{de}}{V_0'l\rho_s}$.

The linear Hasegawa-Mima equation obtains a simplest form

$$\frac{d}{dT}\left\{\left[1+l^2\rho_s^2\left(1+T^2\right)\right]\phi\right\}+iSl\rho_s\phi=0.$$

The solution for $\phi(\tau, k_x, k_y, k_z)$ it is equal to

$$\phi(t, k_x, k_y, k_z) = \phi(t = 0, k_x, k_y, k_z) \frac{1 + \left(k_y^2 \rho_s^2 + k_x^2\right)}{1 + k_y^2 \rho_s^2 + \rho_s^2 \left(k_y V_0' t - k_x\right)^2}$$

$$\times \exp\left\{-i \frac{S}{\sqrt{1 + k_y^2 \rho_s^2}} \left(\tan^{-1} \frac{\rho_s \left(k_y V_0' t - k_x\right)}{\sqrt{1 + k_y^2 \rho_s^2}} + \tan^{-1} \frac{k_x \rho_s}{\sqrt{1 + k_y^2 \rho_s^2}}\right)\right\}.$$

where $S = \frac{k_y v_{de}}{V_0' k_y \rho_s}$.

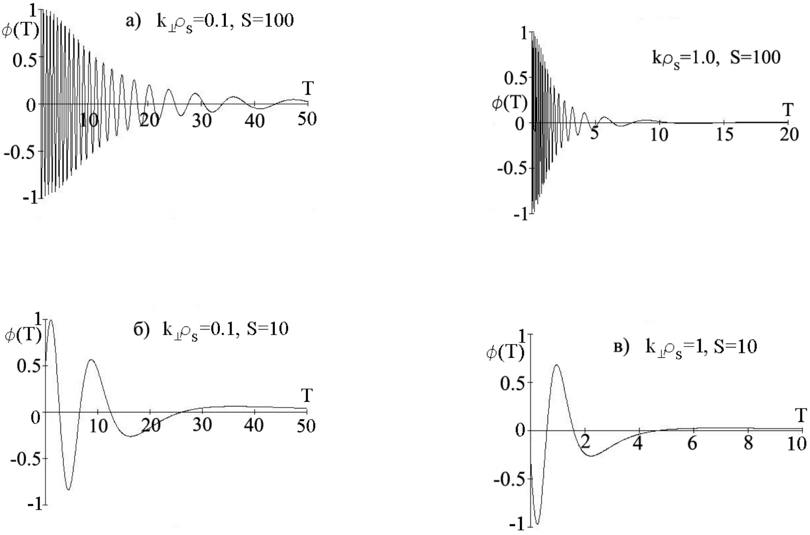
For $V_0' t k_y \rho_s < 1$

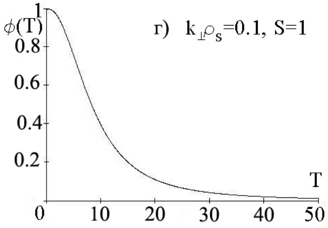
$$\phi(t, k_x, k_y, k_z) \sim \phi(t = 0, k_x, k_y, k_z) e^{i \omega_{de} t};$$

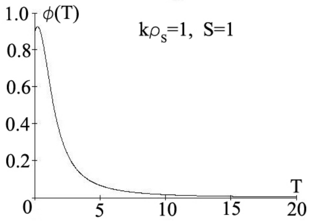
for $V_0' t k_y \rho_s > 1$,

$$\phi(t, k_x, k_y, k_z) \sim \phi(t = 0, k_x, k_y, k_z) \frac{e^{i\alpha}}{\left(k_y \rho_s V_0' t\right)^2}.$$

The suppression of the drift resistive instability in the case of sufficiently strong flow shear is a non–modal process, during which the initial separate spatial Fourier harmonic of the drift wave potential transformed into zero-frequency convective cell with amplitude decreasing with time as $(V_0' t)^{-2}$.


a) $k_{\perp}\rho_s$=0.1, S=100
ϕ(T)
T
$k\rho_s$=1.0, S=100
б) $k_{\perp}\rho_s$=0.1, S=10
в) $k_{\perp}\rho_s$=1, S=10





$$\phi(t) \sim \frac{1}{(V_0' t)^2}$$

RENORMALIZED HYDRODYNAMIC THEORY FOR DRIFT MODES IN PLASMA SHEAR FLOWS

We investigate the temporal evolution of drift modes in time-dependent shear flow using the Hasegawa–Wakatani system of equations for the dimensionless density $n = \tilde{n}/n_e$ and potential $\phi = e\varphi/T_e$ perturbations (n_e is the electron background density, T_e is the electron temperature)

$$\rho_s^2 \left(\frac{\partial}{\partial t} + V_0(x,t) \frac{\partial}{\partial y} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \phi}{\partial x} \frac{\partial}{\partial y} \right) \right) \nabla^2 \phi = a \frac{\partial^2}{\partial z^2} (n - \phi) \, ,$$

$$\left(\frac{\partial}{\partial t} + V_0(x,t) \frac{\partial}{\partial y} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \phi}{\partial x} \frac{\partial}{\partial y} \right) \right) n + v_{de} \frac{\partial \phi}{\partial y} = a \frac{\partial^2}{\partial z^2} (n - \phi) \, .$$

For the homogeneous velocity shear, $V_0' = const$:

$$\rho_s^2 \left(\frac{\partial}{\partial t} + V_0' x \frac{\partial}{\partial y} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \phi}{\partial x} \frac{\partial}{\partial y} \right) \right) \nabla^2 \phi = a \frac{\partial^2}{\partial z^2} (n - \phi) \, ,$$

$$\left(\frac{\partial}{\partial t} + V_0' x \frac{\partial}{\partial y} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \phi}{\partial x} \frac{\partial}{\partial y} \right) \right) n + v_{de} \frac{\partial \phi}{\partial y} = a \frac{\partial^2}{\partial z^2} (n - \phi) \, .$$

Three steps of the renormalization procedure.

The first step.

With new spatial variables ξ, η,

$$t = t, \qquad \xi = x, \qquad \eta = y - V_0' x t, \qquad z = z.$$

the Hasegawa–Wakatani system has a form

$$\rho_s^2 \left(\frac{\partial}{\partial t} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \frac{\partial}{\partial \eta} \right) \right) \Delta \phi = a \frac{\partial^2}{\partial z^2} (n - \phi) \, ,$$

$$\left(\frac{\partial}{\partial t} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \frac{\partial}{\partial \eta} \right) \right) n + v_{de} \frac{\partial \phi}{\partial \eta} = a \frac{\partial^2}{\partial z^2} (n - \phi) \, .$$

The Laplacian operator Δ now becomes time-dependent,

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \left(\frac{\partial}{\partial \xi} - V_0' t \frac{\partial}{\partial \eta} \right) \left(\frac{\partial}{\partial \xi} - V_0' t \frac{\partial}{\partial \eta} \right) + \frac{\partial^2}{\partial \eta^2} .$$

leaving us with an initial value problem to solve.

The conclusions of principal importance:

- With convective variables ξ and η the system of governing equations does not contain any more the spatial dependency connected with the flow shear.
- The linear solution has a form

$$\phi\left(\xi,\eta,t\right)=\int dk_x\int dk_y\phi\left(k_x,k_y,0\right)g\left(k_x,k_y,t\right)e^{ik_x\xi+ik_y\eta},$$

  where $\phi\left(k_x,k_y,0\right)$ is the initial data and $g\left(k_x,k_y,t\right)$ is the linearly unstable solution.
- It was shown (Mikhailenko-2000) for the case of the spatially homogeneous time-independent velocity shear, that for $\omega_d > v_0' \sim \gamma$, where γ is growth rate of the resistive drift instability in plasma without shear flow, the solution $g\left(k_x,k_y,t\right)$ in times $\left(v_0'\right)^{-1} < t \lesssim \left(v_0'k_y\rho_s\right)^{-1}$ still has an ordinary modal form,

$$g\left(k_x,k_y,t\right)=e^{-i\omega_d t+\gamma t}.$$

- In the laboratory frame solution becomes nonseparable in space and time and therefore quite different from the normal mode assumption,

$$\phi\left(\mathbf{r},t\right)=\int dk_x\int dk_y\phi\left(k_x,k_y,0\right)e^{-i\omega_d t+\gamma t+i\left(k_x-k_yv_0't\right)x+ik_yy}$$

It is important to note that the convective nonlinear derivative remains the same in the new convective coordinates as for plasma without any flows.

Therefore, the nonlinear evolution of the resistive drift instability in times $\left(V_0'\right)^{-1} < t < \left(V_0' k_y \rho_s\right)^{-1}$ governed by H–W system will occur as in plasmas without shear flow.

The second step (it is equally valid for plasmas without flows): Transformation to the non-linear convective coordinates.

With new variables ξ_1, η_1,

$$\xi_1 = \xi - \widetilde{\xi}(t) = \xi + \frac{cT_e}{eB}\int_{t_0}^{t} \frac{\partial \phi}{\partial \eta} dt_1, \qquad \eta_1 = \eta - \widetilde{\eta}(t) = \eta - \frac{cT_e}{eB}\int_{t_0}^{t} \frac{\partial \phi}{\partial \xi} dt_1$$

the convective nonlinearity becomes of the higher order with respect to the potential ϕ. Omitting such nonlinearity, as well as small nonlinearity of the second order in the Laplacian, resulted from the transformation to nonlinearly determined variables ξ_1, η_1, we get linear equation with solution, where wave numbers k_x, k_y are conjugate there to coordinates ξ_1, η_1 respectively. With variables ξ and η this solution has a form

$$\phi(\xi, \eta, t) = \int dk_x \int dk_y \phi(k_x, k_y, 0)\, g(k_x, k_y, t_1)\, e^{ik_x \xi_1 + ik_y \eta_1}$$

$$= \int dk_x \int dk_y \phi(k_x, k_y, 0)\, g(k_x, k_y, t_1)\, e^{ik_x \xi + ik_y \eta - ik_x \widetilde{\xi}(t_1) - ik_y \widetilde{\eta}(t_1)},$$

This equation is in fact a nonlinear integral equation for potential ϕ, in which the effect of the total Fourier spectrum on any separate Fourier harmonic is accounted for.

The third step: The calculation the correlations of the plasma displacements in the unstable electric field of the drift turbulence.

Assuming that the displacements $\widetilde{\xi}\,(t)$, $\widetilde{\eta}\,(t)$ obey the Gaussian statistics with mean zero,

$$\left\langle \exp\left[ik_{1x}\left(\widetilde{\xi}\,(t_1)-\widetilde{\xi}\,(t_2)\right)+ik_{1y}\left(\widetilde{\eta}\,(t_1)-\widetilde{\eta}\,(t_2)\right)\right]\right\rangle$$
$$=\exp\left[-\frac{1}{2}k_x^2K_{\xi\xi}\,(t_1,t_2)-k_xk_yK_{\xi\eta}\,(t_1,t_2)-\frac{1}{2}k_y^2K_{\eta\eta}\,(t_1,t_2)\right]$$

we find in this case the following relation for $K_{\xi\xi}\,(t)$:

$$\left\langle\left(\widetilde{\xi}\,(t)\right)^2\right\rangle=K_{\xi\xi}\,(t,t_0)=K_{\xi\xi}\,(t)=\frac{c^2T_e^2}{e^2B^2}\int\limits_{t_0}^{t}dt_1\int\limits_{t_0}^{t}dt_2\int dk_x\int dk_y\,|\phi\,(k_x,k_y,0)|^2\,k_y^2$$
$$\times\exp\left(\gamma\,(t_1+t_2)-i\omega_d\,(t_1-t_2)\right)$$
$$\times\exp\left[-\frac{1}{2}k_x^2K_{\xi\xi}\,(t_1,t_2)-k_xk_yK_{\xi\eta}\,(t_1,t_2)-\frac{1}{2}k_y^2K_{\eta\eta}\,(t_1,t_2)\right].$$

where

$$K_{\xi\xi}\,(t_1,t_2)=\left\langle\left(\widetilde{\xi}\,(t_1)-\widetilde{\xi}\,(t_2)\right)^2\right\rangle,\;K_{\eta\eta}\,(t_1,t_2)=\left\langle\left(\widetilde{\eta}\,(t_1)-\widetilde{\eta}\,(t_2)\right)^2\right\rangle$$
$$K_{\xi\eta}\,(t_1,t_2)=\left\langle\left(\widetilde{\xi}\,(t_1)-\widetilde{\xi}\,(t_2)\right)\left(\widetilde{\eta}\,(t_1)-\widetilde{\eta}\,(t_2)\right)\right\rangle,$$

The two-time scale procedure with time variables $\tau = t_1 - t_2$, $\widehat{t} = (t_1 + t_2)/2$ of the calculation of the dispersion tensor of random displacements of the plasma is developed.

A general equation,

$$k^{2x} K_{\xi\xi}(t) + 2k_x k_y K_{\xi\eta}(t) + k_y^2 K_{\eta\eta}(t)$$

$$= \frac{T_e^2 c^2}{e^2 B^2} \int dk_{1x} \int dk_{1y} \int_0^t d\widehat{t} \left|\phi\left(k_{1x}, k_{1y}, \widehat{t}\,\right)\right|^2 \left|\left[\vec{k}_\perp \times \vec{k}_{1\perp}\right]\right|^2 \frac{C\left(k_{1x}, k_{1y}, \widehat{t}\right)}{\omega_d^2\left(k_{1x}, k_{1y}\right)}$$

$$= 2 \int_{t_0}^t d\widehat{t}\, C\left(k_x, k_y, \widehat{t}\right)$$

where $\left|\phi\left(k_{1x}, k_{1y}, \widehat{t}\,\right)\right|^2 = \left|\phi\left(k_{1x}, k_{1y}, 0\,\right)\right|^2 e^{2\gamma\left(k_{1x}, k_{1y}\right)\widehat{t}}$.

The renormalized form of the potential, in which the average effect of the random convection is accounted for,

$$\phi(\xi, \eta, t) = \int dk_x \int dk_y \phi(k_x, k_y, 0)$$

$$\times \exp\left[-i\omega_d t + \gamma t - \int_{t_0}^t d\widehat{t}\, C\left(k_x,\, k_y,\, \widehat{t}\right) + ik_x\xi + ik_y\eta\right].$$

The saturation of the instability occurs when $\partial\left(\phi\left(\xi,\eta,t\right)\right)^2/\partial t=0$, i.e. when

$$\gamma\left(k_x,k_y\right)=C\left(k_x,k_y,t\right)$$
$$=\frac{T_e^2c^2}{e^2B^2}\int dk_{1x}\int dk_{1y}\left|\phi\left(k_{1x},k_{1y},t\right)\right|^2\left|\left[\vec{k}_\perp\times\vec{k}_{1\perp}\right]\right|^2\frac{C\left(k_{1x},\,k_{1y},\,t\right)}{\omega_d^2\left(k_{1x},k_{1y}\right)}$$

$$\gamma\left(k_x,k_y\right)=\frac{T_e^2c^2}{e^2B^2}\int dk_{1x}\int dk_{1y}\left|\phi\left(k_{1x},k_{1y},t\right)\right|^2\left|\left[\vec{k}_\perp\times\vec{k}_{1\perp}\right]\right|^2\frac{\gamma\left(k_{1x},k_{1y}\right)}{\omega_d^2\left(k_{1x},k_{1y}\right)}$$

The sought–for value is a time t_{sat} at which the balance of the linear growth and nonlinear damping occurs for given initial disturbance $\phi\left(k_{1x},k_{1y},0\right)$ and dispersion. With obtained t_{sat} the saturation level will be equal to $\left|\phi\left(t_{sat}\right)\right|^2$.
The well known order of value estimate for the potential ϕ in the saturation state is obtained easily

$$\frac{e\phi}{T_e}\sim\frac{1}{k_\perp L_n}$$

for times $\left(V_0'\right)^{-1}<t<\left(V_0'k_y\rho_s\right)^{-1}$.
Obtained results show that the nonlinearity of the Hasegawa–Wakatani system of equations in variables ξ and η **does not display any effects of the enhanced decorrelations provided by flow shear**.

In the laboratory frame of reference such spatial Fourier modes are observed as a sheared modes with time dependent component of the wave number $k_{x(lab)} = k_x - k_y v_0' t$, directed along the velocity shear,

$$\phi\left(\mathbf{r},t\right)=\int dk_x\int dk_y\phi\left(k_x,k_y,0\right)e^{i(k_x-k_yv_0't)x+ik_yy-i\omega_dt+\gamma t-ik_x\widetilde{\xi}(t_1)-ik_y\widetilde{\eta}(t_1)}$$

The displacements $\widetilde{\xi}\left(t\right)$ and $\widetilde{\eta}\left(t\right)$ are observed in the laboratory frame as the displacements $\widetilde{x}\left(t\right)$ and $\widetilde{y}\left(t\right)$ which are equal to

$$\widetilde{x}\left(t\right)=\widetilde{\xi}\left(t\right)$$

and

$$\widetilde{y}\left(t\right)=\int\limits_{t_0}^{t}\tilde{v_y}\left(t_1\right)dt_1=\frac{cT_e}{eB}\int\limits_{t_0}^{t}dt_1\frac{\partial\phi}{\partial x}+\int\limits_{\mathbf{t_0}}^{\mathbf{t}}\mathbf{dt_1}\frac{\partial\mathbf{V_0}\left(\mathbf{x},\mathbf{t_1}\right)}{\partial\mathbf{x}}\widetilde{\mathbf{x}}\left(\mathbf{t_1}\right)$$

$$=i\frac{c}{B}\int dk_x\int dk_y\int\limits_{t_0}^{t}dt_1\phi\left(k_x,k_y,0\right)\left(k_x-k_yv_0'\left(t-t_1\right)\right)$$

$$\times\exp\left(-i\omega_dt_1+\gamma t_1+i(k_x-k_yv_0't_1)x+ik_yy-ik_x\widetilde{\xi}\left(t_1\right)-ik_y\widetilde{\eta}\left(t_1\right)\right)$$

The correlation $K_{yy}(t)$ is

$$K_{yy}(t) = \frac{c^2 T_e^2}{e^2 B^2} Re \int dk_x \int dk_y \left|\phi(k_x, k_y, 0)\right|^2 \frac{C_1\left(k_x, k_y, \widehat{t}, x\right)}{\left(\omega_d(k_x, k_y) + k_y v_0' x\right)^2}$$
$$\times \left(\frac{2}{3}\left(k_y v_0'\right)^2 t^3 - 2k_x k_y v_0' t^2 + 2k_x^2 t\right),$$

It displays the effect of the anisotropic dispersion conditioned by flow shear, observed in the laboratory frame of reference: dispersion increases much faster along a flow than in the direction of the flow shear.

This effect of the "enhanced decorrelation by flow shear" have nothing in common with "enhanced suppression" of turbulence in shear flows.

The temporal evolution of the resistive pressure-gradient- driven turbulence and anomalous transport in shear flow across the magnetic field

The temporal evolution of the hydrodynamic resistive pressure-gradient- driven mode in a sheared flow is investigated as a solution of the initial value problem by employing the shearing modes approach.

It reveals essential difference of the processes, which occur in the case of the flows with velocity shear less than the growth rate of the instability in the steady plasmas, with processes which occur in the flows with velocity shear larger than the growth rate.

We found that the suppression of the turbulence by a sheared flow occurs only in the flows with velocity shear larger than the growth rate. In this case, the initial value scheme, which does not impose a priori any constraints on the form that solution may take, is necessary for the proper description of the temporal evolution and eventual suppression of the turbulence in a sheared flow.

The governing equations of the present model includes the resistive and pressure gradient driven instabilities

$$\frac{d}{dt}\left(\tilde{A}_{\|} - \frac{c^2}{\omega_{pe}^2}\nabla_\perp^2 \tilde{A}_{\|}\right) + v_{de}\frac{\partial \tilde{A}_{\|}}{\partial y} - \nu_{ei}\frac{c^2}{\omega_{pe}^2}\nabla_\perp^2 \tilde{A}_{\|}$$

$$= -c\frac{\partial \tilde{\phi}}{\partial z} + \frac{c}{en_0}\frac{\partial \tilde{p}_e}{\partial z} + \frac{c}{en_{e0}B_0}\nabla \tilde{A}_{\|}\cdot \mathbf{b}_0 \times \nabla \tilde{p}_e,$$

$$\frac{d\tilde{p}_e}{dt} + en_{e0}\left(v_{de} - v_{Re}\right)\frac{\partial \tilde{\phi}}{\partial y} + v_{Re}\frac{\partial \tilde{p}_e}{\partial y} + \frac{c\Gamma_e T_{e0}}{4\pi e}\left(\frac{\partial}{\partial z} + \frac{1}{B_0}\nabla \tilde{A}_{\|} \times \mathbf{b}_0 \cdot \nabla\right)\nabla_\perp^2 \tilde{A}_{\|} = 0,$$

$$\frac{d\tilde{p}_i}{dt} - en_{i0}\left(v_{di} - v_{Ri}\right)\frac{\partial \tilde{\phi}}{\partial y} + v_{Ri}\frac{\partial \tilde{p}_i}{\partial y} = 0,$$

$$\left(\frac{d}{dt} + v_{di}\frac{\partial}{\partial y}\right)\nabla_\perp^2 \tilde{\phi} = -\frac{c}{en_{i0}B_0}\nabla_\perp\left(\mathbf{b}_0 \times \nabla \tilde{p}_i \cdot \nabla\right)\nabla_\perp \tilde{\phi} - \nabla_\perp\left(\tilde{\mathbf{v}}_{i\|}\cdot\nabla\right)\nabla_\perp \tilde{\phi}$$

$$-\frac{1}{n_0 e\rho_s^2}v_{Re}\frac{\partial \tilde{p}_e}{\partial y} + \frac{T_e}{T_i}\frac{1}{n_0 e\rho_s^2}v_{Re}\frac{\partial \tilde{p}_e}{\partial y} - \frac{v_A^2}{c}\left(\frac{\partial}{\partial z} + \frac{1}{B_0}\nabla \tilde{A}_{\|} \times \mathbf{b}_0 \cdot \nabla\right)\nabla_\perp^2 \tilde{A}_{\|},$$

The operator d/dt is defined for the sheared flow transverse to the magnetic field $\mathbf{b}_0 = B_0\mathbf{b}_0$ (directed along the z axis), with uniform velocity shearing rate, i.e, $\mathbf{v}_0(x) = v_0' x\mathbf{e}_y$, where v_0' is independent of x, as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_0' x\frac{\partial}{\partial y} + \mathbf{v}_E \cdot \nabla$$

with $\mathbf{v}_E = (c/B_0)\,\mathbf{b}_0 \times \nabla\tilde{\phi}$.

In the sheared coordinates

$$t = t\,, \qquad \xi = x\,, \qquad \eta = y - v_0' x t\,, \qquad z = z$$

convected with the sheared flow the linearized system for the nondimensional variables $\phi = e\tilde{\phi}/T_e$, $A_\parallel = e\tilde{A}_\parallel/T_e$, $p_e = \tilde{p}_e/n_{0e}T_e$, $p_i = \tilde{p}_i/n_{0e}T_i$ becomes

$$\frac{\partial}{\partial t}\left(A_\parallel - \frac{c^2}{\omega_{pe}^2}\nabla_\perp^2 A_\parallel\right) + v_{de}\frac{\partial A_\parallel}{\partial \eta} - \nu_{ei}\frac{c^2}{\omega_{pe}^2}\nabla_\perp^2 \tilde{A}_\parallel = -c\frac{\partial \phi}{\partial z} + c\frac{\partial p_e}{\partial z}\,,$$

$$\frac{\partial p_e}{\partial t} + (v_{de} - v_{Re})\frac{\partial \phi}{\partial \eta} + v_{Re}\frac{\partial p_e}{\partial \eta} + \Gamma\frac{c^2}{\omega_{pe}^2}\frac{v_{Te}^2}{c}\frac{\partial}{\partial z}\nabla_\perp^2 A_\parallel = 0,$$

$$\frac{\partial p_i}{\partial t} - (v_{di} - v_{Ri})\frac{\partial \phi}{\partial \eta} + v_{Ri}\frac{\partial p_i}{\partial \eta} = 0\,,$$

$$\left(\frac{\partial}{\partial t} + v_{di}\frac{\partial}{\partial \eta}\right)\nabla_\perp^2 \phi = v_0'\frac{\partial}{\partial \eta}\left(\frac{\partial p_i}{\partial \xi} - v_0' t\frac{\partial p_i}{\partial \eta}\right)$$
$$-\frac{1}{\rho_s^2}v_{Re}\frac{\partial \tilde{p}_e}{\partial \eta} + \frac{T_e}{T_i}\frac{1}{\rho_s^2}v_{Re}\frac{\partial \tilde{p}_e}{\partial \eta} - \frac{v_A^2}{c}\frac{\partial}{\partial z}\nabla_\perp^2 A_\parallel\,,$$

where the operator $\nabla_\perp^2$ is

$$\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial \eta^2} + \left(\frac{\partial}{\partial \xi} - v_0' t\frac{\partial}{\partial \eta}\right)^2\,.$$

With new coordinates the spatial inhomogeneity introduced by the sheared flow velocity is excluded from the system and the effects of the flow shearing are transferred to the time domain.

$$A_{\|}(t, k_\perp, l, k_z) = \iiint d\xi\, d\eta\, dz\, A_{\|}(t, \xi, \eta, z) \exp\{-ik_\perp \xi - il\eta - ik_z z\} \;,$$

and the solution of the Fourier transformed system reduces to the initial value problem of the temporal evolution of the separate spatial Fourier harmonics of $A_{\|}$, ϕ, p_e and p_i.

$$\frac{dg}{dT} + F(T)\, g = 0.$$

$$g = (\Psi, p_e, p_i, U), \quad \Psi = \left(1 + \frac{c^2 l^2}{\omega_{pe}^2}\left(1 + T^2\right)\right) A_{\|}, \quad U = \left(1 + T^2\right) \phi$$

$$F(T) = \begin{pmatrix} \frac{iC_e + q^2 R_e\left(1+T^2\right)}{1+q^2\left(1+T^2\right)} & -i\frac{c}{v_A}S & 0 & i\frac{c}{v_A}S\frac{1}{1+T^2} \\ -\frac{iS\frac{v_A}{c}l^2\rho_s^2\left(1+T^2\right)}{1+q^2\left(1+T^2\right)} & iQ_e & 0 & i\left(C_e - Q_e\right)\frac{1}{1+T^2} \\ 0 & 0 & iQ_i & -i\left(C_i - Q_i\right)\frac{T_e}{T_i}\frac{1}{1+T^2} \\ \frac{i\frac{v_A}{c}S\left(1+T^2\right)}{1+q^2\left(1+T^2\right)} & -\frac{iQ_e}{l^2\rho_s^2} & T\frac{T_i}{T_e} + \frac{iQ_i}{l^2\rho_s^2} & iC_i \end{pmatrix}.$$

$T = v_0' t - \frac{k_\perp}{l}$, $\rho_s = v_s/\omega_{ci} = (T_e/T_i)^{1/2}\,\rho_i$ is the ion-sound Larmor radius, $v_s = (\Gamma_e T_e/m_i)^{1/2}$ is the ion-sound velocity, ρ_i is the ion thermal Larmor radius, $v_{Te(Ti)}$ is the electron (ion) thermal velocity, and v_A is the Alfvén velocity.

$$S = \frac{k_z v_A}{v_0'}, \quad C_{e,i} = \frac{l v_{de,di}}{v_0'},$$

$$Q_{e,i} = \frac{l v_{Re,Ri}}{v_0'}, \quad R_e = \frac{\nu_{ei}}{v_0'}, \quad q^2 = \frac{c^2 l^2}{\omega_{pe}^2} = \frac{m_e}{m_i}\frac{1}{\beta}l^2\rho_s^2$$

The "almost" modal solutions exist in the time interval

$$\left(v_0'\right)^{-1} \gg t \gtrsim \left(l v_{de}\right)^{-1}, \left(k_z v_A\right)^{-1},$$

i.e. for large values of the dimensionless parameters

$$S \gg 1, \quad S/C_{e(i)} = \mathcal{O}(1), \quad S/Q_{e(i)} = \mathcal{O}(1), \quad S/R_e = \mathcal{O}(1).$$

This is a case of a weak flow shear.

The time dependence of $F(T)$ becomes strong for $T > 1$, i.e. for time $t > \left(v_0'\right)^{-1}$. At this time the non-modal structure of the solution occurs and the solution of the initial value problem becomes necessary.

For time $t \gtrsim \left(l v_{de}\right)^{-1}, \left(k_z v_A\right)^{-1}$ the condition $t \gg \left(v_0'\right)^{-1}$ corresponds to small values of the dimensionless parameters

$$S \ll 1, \quad S/C_{e(i)} = \mathcal{O}(1), \quad S/Q_{e(i)} = \mathcal{O}(1), \quad S/R_e = \mathcal{O}(1) \text{ and } T > 1.$$

This is a case of a strong flow shear.

Nonmodal linear analysis for a weak flow shear

$S \to \lambda S$, $C_{e(i)} \to \lambda C_{e(i)}$, $Q_{e(i)} \to \lambda Q_{e(i)}$, $R_e \to \lambda R_e$,
The regime of weak flow shear corresponds to large values of the parameter λ.

$$g\left(T,\lambda\right) = a\left(T\right)\exp\left(-i\lambda\int\limits_{T_0}^{T}\Omega\left(T_1,\lambda\right)dT_1\right)$$

where $a(T)$ is a column-vector, and $\Omega(T,\lambda) = \sum_{i=0}^{\infty}\Omega_i(T)\lambda^{-i}$.
For $\omega\left(t\right) = v_0'\Omega_0(T)$:

$$\begin{gathered}\left(\omega\left(t\right) - lv_{de} + i\frac{m_e}{m_i}\frac{\nu_{ei}}{\beta}K_\perp^2\left(t\right)\rho_s^2\right) \\ \times\left[\left(\omega\left(t\right) - lv_{Re}\right)\left(\omega\left(t\right) - lv_{di}\right)\left(\omega\left(t\right) - lv_{Ri}\right)K_\perp^2\left(t\right)\rho_s^2\right. \\ \left.-\omega\left(t\right)\left(lv_{de} - lv_{Re}\right)\left(lv_{Ri} - lv_{Re}\right)\right] \\ = k_z^2 v_A^2 K_\perp^2\left(t\right)\rho_s^2\left[K_\perp^2\left(t\right)\rho_s^2\left(\omega\left(t\right) - lv_{di}\right)\left(\omega\left(t\right) - lv_{Ri}\right)\right. \\ \left.+\omega\left(t\right)\left(\omega\left(t\right) - lv_{de} + lv_{Re} - lv_{Ri}\right)\right] = 0,\end{gathered}$$

and $K_\perp^2\left(t\right) = l^2\left(1 + T^2\right)$.

The resistive drift – Alfven instability.

The case of a spatially homogeneous magnetic field, i.e. when $v_{Re} = v_{Ri} = 0$.

$$\left(\omega\left(t\right) - lv_{de}\right)\left(\omega^2\left(t\right) - \omega\left(t\right)lv_{di} - k_z^2 v_A^2\right)$$
$$= K_\perp^2\left(t\right)\rho_s^2 k_z^2 v_A^2\left(\omega\left(t\right) - lv_{di}\right)\left(1 - i\nu_{ei}\frac{\omega\left(t\right)}{k_z^2 v_{Te}^2}\right),$$

describes the long wave length, $K_\perp\left(t\right)\rho_s \ll 1$, resistive drift – Alfven instability of steady plasmas.

$\omega_{01} = lv_{de}$, $\omega_{02,03} = lv_{di}/2 \pm \left(l^2 v_{di}^2/4 + k_z^2 v_A^2\right)^{1/4}$. The the drift wave ω_{01} and shear Alfven wave ω_{02} are coupled due to the finite ion Larmor radius effect. The instability occurs when $\omega_{01} \simeq \omega_{02}$. The coupling is small when $K_\perp\left(t\right)\rho_s \ll 1$ and reveals in the development of the instability with the growth rate $\gamma\left(t\right) = Im\,\delta\omega\left(t\right)$, where

$$\delta\omega\left(t\right) = \pm K_\perp\left(t\right)\rho_s k_z v_A\left(\frac{\omega_{01} + |lv_{di}|}{\omega_{02} - \omega_{03}}\right)\left(1 - i\frac{\nu_{ei}\omega_{01}}{k_z^2 v_{Te}^2}\right)^{1/2}.$$

$$\gamma\left(t\right) \sim \nu_{ei} q\left(\frac{m_e}{m_i\beta}\right)^{1/2} \ll \nu_{ei}.$$

The pressure-gradient-driven Rayleigh – Taylor (RT) instability of a plasma with cold ions

$$\omega(t)\left(\omega(t) - lv_{Re}\right) K_{\perp}^{2}(t)\,\rho_{s}^{2} + lv_{Re}\left(lv_{de} - lv_{Re}\right) = 0.$$

$Re\,\omega(t) = lv_{Re}/2,$

$$\gamma(t) = Im\,\omega(t) = \frac{lv_{Re}}{K_{\perp}(t)\,\rho_{s}}\left(\frac{v_{de}}{v_{Re}} - \left(1 + \frac{K_{\perp}^{2}(t)\,\rho_{s}^{2}}{4}\right)\right)^{1/2} \approx \frac{l\left(v_{Re}v_{de}\right)^{1/2}}{K_{\perp}(t)\,\rho_{s}},$$

The RT instability develops when $v_{de}/v_{Re} > 1$. The qualitative results obtained by the numerical solution of the complete dispersion equation:
1) the maximum growth rate attains for the flute perturbations with $k_z = 0$ and it rapidly decreases when k_z grows;
2) the growth rate is maximum for cold ions with $T_i \ll T_e$; in this case the ion diamagnetic drift and ion drift in the curved magnetic field are negligibly small.

$$\frac{\gamma_{(R)}}{\gamma_{(RT)}} \sim K_{\perp}^{2}\rho_{s}^{2}\,\frac{m_e}{m_i\beta}\,\frac{\nu_{ei}}{l\left(v_{de}v_{Re}\right)^{1/2}} \sim q^{2}\,\frac{\nu_{ei}}{l\left(v_{de}v_{Re}\right)^{1/2}} \ll 1\,,$$

The RT instability has much larger growth rate $\gamma_{(RT)}$ than the growth rate $\gamma_{(R)}$ of the resistive drift–Alfven instability in plasmas with $q \ll 1$.

Nonmodal linear analysis for a strong flow shear

In the shearing flow, the nonmodal effects will determine the long time, $t > \gamma^{-1}$, linear evolution when they develop before the modal nonlinear effects become strong enough. The linear evolution of the instabilities determined by Eq. (16), becomes strongly non-modal in a sheared flow when time $T \gg 1$.

At time $T \gg 1$, two time intervals should be distinguished. In the first interval,

$$q^{-1} > T > 1\,,$$

the terms which contain small parameter q^2 may be neglected in matrix $F\left(T\right)$. For the dimensional time this interval is determined as

$$t_s > \frac{1}{qv_0'} > t > \frac{1}{v_0'}.$$

$t_s \approx \left(v_0' l \rho_i\right)^{-1}$ is a time at which $K_\perp\left(t\right)\rho_i$ becomes equal to unity and the non-modal kinetic approach becomes necessary.

The second interval is determined as

$$T > q^{-1},$$

i.e. for time t

$$t_s > t > \frac{1}{qv_0'} = \left(\frac{T_i}{T_e}\frac{m_i}{m_e}\beta\right)^{1/2} t_s.$$

In this interval, q^2T^2 is above the unity and the terms with q^2T^2 in the matrix $F\left(T\right)$ should be retained. For $\beta > m_i/m_e$ and $T_i \sim T_e$ inequality is not valid.

The non-modal temporal evolution of the flute RT instability is the dominant process in first time interval. The system reduces to two equations

$$\frac{\partial p_e}{\partial T} + iQ_e p_e + i\left(C_e - Q_e\right)\frac{U}{T^2} = 0,$$

$$\frac{\partial U}{\partial T} - i\frac{Q_e}{l^2\rho_s^2}p_e = 0,$$

which transforms into simple equation for the function $G\left(T\right) = e^{-\frac{i}{2}Q_e T}U\left(T\right)$,

$$\frac{\partial^2 G\left(T\right)}{\partial T^2} + Q_e\left(\frac{Q_e}{4} - \frac{C_e - Q_e}{l^2\rho_s^2 T^2}\right)G\left(T\right) = 0.$$

In time interval

$$\frac{2}{l\rho_s}\left(\frac{C_e}{Q_e}\right)^{1/2} > T > 1\,,$$

the solution to this equation is

$$\phi\left(T\right) \approx \phi_1^{(1)}\left(k_\perp, l, T_0\right)e^{-\frac{i}{2}Q_e T}T^{\nu_1} + \phi_2^{(1)}\left(k_\perp, l, T_0\right)e^{-\frac{i}{2}Q_e T}T^{\nu_2},$$

$$\nu_{1,2} = -\frac{3}{2} \pm \left(\frac{1}{4} + \left(\frac{\gamma}{v_0'}\right)^2\right)^{1/2}.$$

The nonmodal damping occurs when the shearing rate $v_0' > \gamma_{(RT)}/\sqrt{2}$, In time $T > \frac{2}{l\rho_s}\left(\frac{C_e}{Q_e}\right)^{1/2}$ above the first interval potential $\phi\left(T\right)$ decays with time as

$$\phi\left(T\right) \approx \phi_1^{(2)}\left(k_\perp, l, T_0\right)T^{-2} + \phi_2^{(2)}\left(k_\perp, l, T_0\right)T^{-2}e^{-iQ_e T},$$

The running diffusion coefficients

$D_\xi\left(t,t_0\right)$ along the ξ direction

$$D_\xi\left(t,t_1\right)=\frac{1}{2}\frac{d}{dt}\left\langle\left[\delta\xi\left(t,t_1\right)\right]^2\right\rangle,\quad \delta\xi\left(t,t_1\right)=\int_{t_1}^{t}dt'v_\xi\left(\xi',\eta',t',t_0\right),$$

The velocity v_ξ in first time interval is

$$v_\xi\left(\xi,\eta,t\right)=-\frac{c}{B_0}\frac{\partial\phi\left(\xi,\eta,t\right)}{\partial\eta}\simeq i\frac{c}{B_0}\int dk_\perp dll e^{ik_\perp\xi+il\eta}e^{-\frac{i}{2}lv_{Re}t}\phi_1^{(1)}\left(k_\perp,l,t_0\right)\left(v_0't\right)^{\nu_1}$$

$$D_\xi\left(t,t_1\right)\approx\frac{c^2}{B_0^2\left(\nu_1+1\right)}\int dk_\perp dll^2\left|\phi_1^{(1)}\left(k_\perp,l,t_0\right)\right|^2\left(v_0'\right)^{2\nu_1}t^{2\nu_1+1}.$$

The eventual suppression of the RT turbulence occurs in the second time interval.

$$D_\xi\left(t,t_1\right)=\frac{c^2}{B_0^2}\frac{1}{\left(v_0'\right)^4}\int dk_\perp dll^2\left|\phi_1^{(2)}\left(k_\perp,l,t_0\right)\right|^2\frac{1}{t^2}\left(\frac{1}{t_1}-\frac{1}{t}\right),$$

The running diffusion coefficients $D_x(t, t_1)$ and $D_y(t, t_1)$ for the laboratory frame.
$\delta x = \delta\xi, \Longrightarrow D_x(t, t_1) = D_\xi(t, t_1)$.

$$dy(t) = v_y(t)\,dt = d\eta(t) + v_0(\xi(t))\,dt = v_\eta(t)\,dt + v_0'\xi(t)\,dt,$$

$$D_y(t, t_1) = \frac{1}{2}\frac{d}{dt}\left\langle[\delta y(t, t_1)]^2\right\rangle$$
$$\approx \frac{c^2}{B_0^2}\int dk_\perp dll^2 \left|\phi_1^{(1)}(k_\perp, l, t_0)\right|^2 \frac{v_0'(v_0't)^{2\nu_1+3}}{(\nu_1+1)^2(\nu_1+2)}.$$

In the second interval, where $|v_0't| > |v_0't_1| \gg 1$,

$$\left\langle[\delta y(t, t_1)]^2\right\rangle = \frac{c^2}{B_0^2}\int dk_\perp dl \left|\phi_1^{(1)}(k_\perp, l, t_0)\right|^2 \frac{l^2t^2}{(v_0't_1)^2}$$

- the result pertinent for the deterministic motion, that $\left\langle[\delta y(t, t_1)]^2\right\rangle^{1/2} \sim t$. The running diffusion-convection coefficient $D_y(t, t_1)$:

$$D_y(t, t_1) \approx \frac{c^2}{B_0^2}\int dk_\perp dll^2 \left|\phi_1^{(2)}(k_\perp, l, t_0)\right|^2 \frac{t}{(v_0't_1)^2}.$$

In the second time interval the transport of plasma along the sheared flow observed in the laboratory frame is almost convective.

Shearing modes approach to the kinetic theory of plasma shear flows.

$$\frac{\partial F_\alpha}{\partial t} + \hat{\mathbf{v}}\frac{\partial F_\alpha}{\partial \hat{\mathbf{r}}} + \frac{e}{m_\alpha}\left(\mathbf{E}_0\left(\hat{\mathbf{r}}\right) + \frac{1}{c}\left[\hat{\mathbf{v}}\times\mathbf{B}\right] - \bigtriangledown\varphi\left(\hat{\mathbf{r}},t\right)\right)\frac{\partial F_\alpha}{\partial \hat{\mathbf{v}}} = 0,$$

$$\triangle\,\varphi\left(\hat{\mathbf{r}},t\right) = -4\pi\sum_{\alpha=i,e} e_\alpha \int f_\alpha\left(\hat{\mathbf{v}},\hat{\mathbf{r}},t\right)d\hat{\mathbf{v}}.$$

USUALLY only the transformation to the convected coordinates in velocity space,

$$\hat{v}_x = v_x,\;\; \hat{v}_y = v_y + V_0(x),\;\; \hat{v}_z = v_z,$$

without the transformation to the sheared coordinates in the configuration space
was used in the kinetic theory of plasma shear flows. After such transformation the linearized Vlasov equation becomes

$$\frac{\partial f_\alpha}{\partial t} + V_0\left(\hat{x}\right)\frac{\partial f_\alpha}{\partial \hat{y}} + \hat{\mathbf{v}}\frac{\partial f_\alpha}{\partial \hat{\mathbf{r}}} + \omega_{c\alpha}v_{\alpha y}\frac{\partial f_\alpha}{\partial v_{\alpha x}} - \left(\omega_{c\alpha} + V'_\alpha\right)v_{\alpha x}\frac{\partial f_\alpha}{\partial v_{\alpha y}}$$
$$= \frac{e_\alpha}{m_\alpha}\bigtriangledown\varphi\left(\hat{\mathbf{r}},t\right)\frac{\partial F_{0\alpha}}{\partial \hat{\mathbf{v}}}.$$

The approximation of the "slow" spatial variation of the flow velocity $\hat{\omega} = \omega + k_y V_0\left(x\right)$.

Vlasov equation in convected–sheared coordinates

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For $\mathbf{V}_0(\mathbf{r}) = V_0(\hat{x})\,\mathbf{e}_y = V_0'\hat{x}\mathbf{e}_y$

Convected coordinates: $\hat{v}_x = v_x, \qquad \hat{v}_y = v_y + V_0'x, \qquad \hat{v}_z = v_z$

Sheared coordinates: $\hat{x} = x, \qquad \hat{y} = y + V_0'tx, \qquad \hat{z} = z$

$$\frac{\partial f_\alpha}{\partial t} + v_{\alpha x}\frac{\partial f_\alpha}{\partial x} + \left(v_{\alpha y} - v_{\alpha x}V_\alpha' t\right)\frac{\partial f_\alpha}{\partial y} + v_{\alpha z}\frac{\partial f_\alpha}{\partial z_\alpha} +$$
$$\omega_{c\alpha}v_{\alpha y}\frac{\partial f_\alpha}{\partial v_{\alpha x}} - \left(\omega_{c\alpha} + V_\alpha'\right)v_{\alpha x}\frac{\partial f_\alpha}{\partial v_{\alpha y}}$$
$$= \frac{e_\alpha}{m_\alpha}\left(\frac{\partial \varphi}{\partial x} - V_\alpha' t\frac{\partial \varphi}{\partial y}\right)\frac{\partial F_{0\alpha}}{\partial v_{\alpha x}} + \frac{e_\alpha}{m_\alpha}\frac{\partial \varphi}{\partial y}\frac{\partial F_{0\alpha}}{\partial v_{\alpha y}} + \frac{e_\alpha}{m_\alpha}\frac{\partial \varphi}{\partial z_\alpha}\frac{\partial F_{0\alpha}}{\partial v_{\alpha z}}.$$

The spatial inhomogeneity originated from the sheared flow velocity is excluded completely. Now, spatial Fourier transform may be performed.

The Fourier transform in sheared coordinates

$$\varphi(\mathbf{r},t)=\int\varphi(\mathbf{k},t)\,e^{ik_xx+ik_yy+ik_zz}d\mathbf{k},$$

$$\frac{\partial f_\alpha}{\partial t}+\left(i\left(k_x-V_0'tk_y\right)v_{\alpha x}+ik_yv_{\alpha y}+ik_zv_{\alpha z}\right)f_\alpha\left(\mathbf{v}_\alpha,\mathbf{k},t\right)$$
$$+\omega_{c\alpha}v_{\alpha y}\frac{\partial f_\alpha}{\partial v_{\alpha x}}-\left(\omega_{c\alpha}+V_\alpha'\right)v_{\alpha x}\frac{\partial f_\alpha}{\partial v_{\alpha y}}$$
$$=i\frac{e_\alpha}{m_\alpha}\varphi(\mathbf{k},t)\left[\left(k_x-V_0'tk_y\right)\frac{\partial F_{0\alpha}}{\partial v_{\alpha x}}+ik_y\frac{\partial F_{0\alpha}}{\partial v_{\alpha y}}+ik_z\frac{\partial F_{0\alpha}}{\partial v_{\alpha z}}\right].$$

In the laboratory set of references, it becomes a shearing mode

$$\varphi(\hat{\mathbf{r}},t)=\int\varphi(\mathbf{k},t)\,e^{ik_x\hat{x}+ik_y\left(\hat{y}-V_0't\hat{x}\right)+ik_z\hat{z}}d\mathbf{k}$$
$$=\int\varphi(\mathbf{k},t)\,e^{i\left(k_x-V_0'tk_y\right)\hat{x}+ik_y\hat{y}+ik_z\hat{z}}d\mathbf{k}.$$

with time dependent wave number $k_x-V_0'tk_y$.

The solution of the Vlasov equation in the form of the separate Fourier harmonic with time independent wave numbers may be obtained only in convected-sheared coordinates.

This solution reveals in the laboratory frame as a shearing mode with time dependent x–component of the wave number.

With new leading center coordinates X, Y,

$$x = X - \frac{v_\perp}{\sqrt{\eta}\omega_c}\sin\phi, \qquad y = Y + \frac{v_\perp}{\eta\omega_c}\cos\phi + V_0' t \ (X - x),$$

$$z_1 = z - v_z t, \qquad \eta = 1 + \frac{V_0'}{\omega_{ci}},$$

the Vlasov equation transforms into the form

$$\frac{\partial F}{\partial t} + \frac{e}{m\sqrt{\eta}\omega_c}\left(\frac{\partial \varphi}{\partial X}\frac{\partial F}{\partial Y} - \frac{\partial \varphi}{\partial Y}\frac{\partial F}{\partial X}\right)$$
$$+\frac{e}{m}\frac{\sqrt{\eta}\omega_c}{v_\perp}\left(\frac{\partial \varphi}{\partial \phi_1}\frac{\partial F}{\partial v_\perp} - \frac{\partial \varphi}{\partial v_\perp}\frac{\partial F}{\partial \phi_1}\right) - \frac{e}{m}\frac{\partial \varphi}{\partial z_1}\frac{\partial F}{\partial v_z} = 0,$$

$$f = \frac{e}{m}\int\limits_{t_o}^{t}\left[\frac{1}{\omega_c}\frac{\partial \varphi}{\partial Y}\frac{\partial F_0}{\partial X} - \frac{\omega_c}{v_\perp}\frac{\partial \varphi}{\partial \phi_1}\frac{\partial F_0}{\partial v_\perp} + \frac{\partial \varphi}{\partial z_1}\frac{\partial F_0}{\partial v_z}\right]dt'.$$

With leading center convected-sheared coordinates the Fourier transformation of the perturbed potential becomes

$$
\begin{aligned}
\varphi(\mathbf{r},t) &= \int \varphi(t,\mathbf{k}) \exp(ik_x x + ik_y y + ik_z)\, d\mathbf{k} \\
&= \int \varphi(t,\mathbf{k}) \exp(ik_x X + ik_y Y + ik_z z) \\
&\times \exp\left[-\frac{ik_\perp(t)\, v_\perp}{\sqrt{\eta}\omega_c} \sin\left(\phi_1 - \sqrt{\eta}\omega_c t - \theta(t)\right)\right] d\mathbf{k}
\end{aligned}
$$

where $k_\perp^2(t) = \left(k_x - V_0' t k_y\right)^2 + \frac{1}{\eta} k_y^2$, and $\tan\theta = k_y/\sqrt{\eta}(k_x - V_0' t k_y)$.

- **The time dependence of the finite ion-Larmor-radius-effect is the basic linear mechanism of the influence of the velocity shear on waves and instabilities in plasma shear flow.**

- **Larmor radius effect here reveals the interaction of the perturbation having time independent wave numbers k_x, k_y, k_z with the ion, whose Larmor orbit is observed in sheared coordinates as being subjected to persistent stretching.**

- **This effect appears analytically identical to the interaction of the perturbation with time-dependent wave numbers $k_x - V_0' t k_y$, k_y, k_z, the ion orbiting elliptically in the convecting frame.**

- **Drift kinetic equation does not contains the shearing flow effects.**

$$f\left(t,X,Y,v_\perp,\phi,v_z,z_1\right)=\frac{ie}{m}\sum_{n=-\infty}^{\infty}\sum_{n_1=-\infty}^{\infty}\int^{t}dt_1\varphi\left(t_1,\mathbf{k}\right)$$

$$\times\exp\Big(-ik_zv_z\left(t-t_1\right)+in\left(\phi_1-\sqrt{\eta}\omega_ct-\theta\left(t\right)\right)$$

$$-in_1\left(\phi_1-\sqrt{\eta}\omega_ct_1-\theta\left(t\right)\right)\Big)J_n\left(\frac{k_\perp\left(t\right)v_\perp}{\sqrt{\eta}\omega_c}\right)J_{n_1}\left(\frac{k_\perp\left(t_1\right)v_\perp}{\sqrt{\eta}\omega_c}\right)$$

$$\times\left[\frac{k_y}{\eta\omega_{c\alpha}}\frac{\partial F_\alpha}{\partial X_\alpha}+\frac{\sqrt{\eta}\omega_cn_1}{v_\perp}\frac{\partial F_\alpha}{\partial v_\perp}+k_{1z}\frac{\partial F_\alpha}{\partial v_z}\right]$$

$$\left[\left(k_x-v_0'tk_y\right)^2+k_y^2+k_z^2\right]\varphi\left(\mathbf{k},t\right)=\sum_\alpha\frac{i}{\lambda_{D\alpha}^2}\sum_{n=-\infty}^{\infty}\int_{t_0}^{t}dt_1\varphi\left(\mathbf{k},t_1\right)$$

$$\times I_n\left(k_\perp\left(t\right)k_\perp\left(t_1\right)\rho_\alpha^2\right)\exp\left[-\frac{1}{2}\rho_\alpha^2\left(k_\perp^2\left(t\right)+k_\perp^2\left(t_1\right)\right)\right]$$

$$\times\exp\left[-\frac{1}{2}k_z^2v_{T\alpha}^2\left(t-t_1\right)^2-in\sqrt{\eta}\omega_{c\alpha}\left(t-t_1\right)-in\left(\theta\left(t\right)-\theta\left(t_1\right)\right)\right]$$

$$\times\left[\frac{k_yv_{d\alpha}}{\sqrt{\eta}}-n\sqrt{\eta}\omega_{c\alpha}+ik_z^2v_{T\alpha}^2\left(t-t_1\right)\right]+\sum_\alpha e_\alpha\delta n_\alpha\left(\mathbf{k},t_0\right)$$

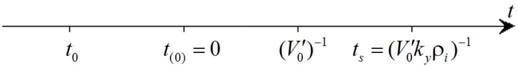

Figure 1: The sequence of the characteristic times for the long wavelength perturbations with $\hat{k}_\perp(t_0)\,\rho_i < 1$.

$$\varphi(\mathbf{k}, t) = \varphi_0 \exp\left[-i\omega(\mathbf{k})\, t \left(1 - \frac{(1+\tau)}{a_i b_i} \frac{t^2}{3t_s^2} \Theta(t)\right) + \gamma(\mathbf{k})\, t - \frac{t^2 \Theta(t)}{2a_i t_s^2}\right].$$

where $a_i = \tau + k_\perp^2 \rho_i^2$, $b_i = 1 - k_\perp^2 \rho_i^2$, $\quad t_s = \left(V_0' k_y \rho_i\right)^{-1}$,

$\Theta(t) = 0$ for $t < 0$ and $\Theta(t) = 1$ for $t \geq 0$.

The analysis on the base of drift kinetic equation

$$\frac{\partial f}{\partial t}+\left(\mathbf{v}_{\|}+\frac{c}{B}\left[\mathbf{E}_{0}\times\mathbf{n}\right]\right)\frac{\partial f}{\partial\mathbf{R}}=\frac{c}{B}\left[\nabla\Phi\times\mathbf{n}\right]\frac{\partial F}{\partial\mathbf{R}}+e\left(\mathbf{v}\cdot\nabla\Phi\right)\frac{\partial F}{\partial E}$$

where

$$E=\frac{mv^{2}}{2}+e\Phi\left(\mathbf{r},t\right),\qquad\mathbf{E}_{0}\left(\mathbf{r}\right)=E_{0}^{\prime}x\mathbf{e}_{x}$$

With convective coordinates $x=\xi$, $y=\eta+V_{0}^{\prime}\xi t$ drift kinetic equations with Poisson equation give for Maxwellian distribution F the integral equation for the potential Φ,

$$-k^{2}\left(t\right)\Phi\left(\mathbf{k},t\right)+\Phi\left(\mathbf{k},t\right)\sum_{\alpha}\frac{1}{\lambda_{D\alpha}^{2}}$$

$$+\sum_{\alpha}\frac{1}{\lambda_{D\alpha}^{2}}\int_{-\infty}^{t}dt_{1}\exp\left(-\frac{k_{z}^{2}v_{T\alpha}^{2}}{2}\left(t-t_{1}\right)^{2}\right)\left(ik_{y}v_{d\alpha}\Phi\left(\mathbf{k},t_{1}\right)+\frac{d\Phi\left(\mathbf{k},t_{1}\right)}{dt_{1}}\right)=0$$

where $k^{2}\left(t\right)=k_{\eta}^{2}+\left(k_{\xi}-V_{0}^{\prime}tk_{\eta}\right)^{2}+k_{z}^{2}$.

$$\int_{-\infty}^{\infty} dte^{i\omega t}\Phi\left(\mathbf{k},t\right)\left(-2\frac{k_\xi k_\eta}{k_0^2}V_0't+\frac{k_\eta^2}{k_0^2}\left(V_0't\right)^2\right)+\Phi\left(\mathbf{k},\omega\right)\varepsilon\left(\mathbf{k},\omega\right)=0$$

where $k_0^2=k_\xi^2+k_\eta^2+k_z^2$ and

$$\varepsilon\left(\mathbf{k},\omega\right)=1+\sum_\alpha\frac{1}{\lambda_{D\alpha}^2}\left(1+i\frac{\left(\omega-k_yv_{d\alpha}\right)}{\sqrt{2}k_zv_{T\alpha}}W\left(\frac{\omega}{2k_zv_{T\alpha}}\right)\right)$$

Without shear flow we have known equation $\Phi\left(\mathbf{k},\omega\right)\varepsilon\left(\mathbf{k},\omega\right)=0$ with solution $\omega=\omega\left(\mathbf{k}\right)$. The presence of shear flow modifies that solution. Assuming, that $k_0^2\lambda_{D\alpha}^2\left(V_0't\right)^2\ll 1$,

$$\Phi\left(\mathbf{k},t\right)=C\exp\left(-i\omega\left(\mathbf{k}\right)t-t\left(\frac{\partial\varepsilon}{\partial\omega\left(\mathbf{k}_0\right)}\right)^{-1}\left(-\frac{k_\xi k_\eta}{k_0^2}V_0't+\frac{1}{3}\frac{k_\eta^2}{k_0^2}\left(V_0't\right)^2\right)\right)$$

The linear drift kinetic equation incorporate only one effect conditioned by shear flow. It is the reduction of the waves frequencies. For drift waves with $k_zv_{ti}\ll\omega\left(\mathbf{k}_0\right)\ll k_zv_{te}$,

$$\Phi\left(\mathbf{k},t\right)=C\exp\left(-ik_yv_{de}t\left(1-\frac{k_\eta^2v_s^2}{\omega_{pi}^2}\left(-\frac{k_\xi}{k_\eta}V_0't+\frac{1}{3}\left(V_0't\right)^2\right)\right)\right).$$

Renormalized Non–Modal Theory of Drift Turbulence in Plasma Shear Flows

$$
\begin{aligned}
dt &= -\left(\frac{e}{\sqrt{\eta_i} m_i \omega_{ci}} \frac{\partial \varphi}{\partial Y_1}\right)^{-1} dX = \left(\frac{e}{\sqrt{\eta_i} m_i \omega_{ci}} \frac{\partial \varphi}{\partial X_1}\right)^{-1} dY \\
&= \left(\frac{e}{m_i} \frac{\sqrt{\eta_i}\omega_{ci}}{v_\perp} \frac{\partial \varphi}{\partial \phi_1}\right)^{-1} dv_\perp \\
&= -\left(\frac{e}{m_i} \frac{\sqrt{\eta_i}\omega_{ci}}{v_\perp} \frac{\partial \varphi}{\partial v_\perp}\right)^{-1} d\phi_1 = -\left(\frac{e}{m_i} \frac{\partial \varphi}{\partial z_1}\right) dv_z \\
&= \left(\frac{e}{\sqrt{\eta_i} m_i \omega_{ci}} \frac{\partial \varphi}{\partial Y} \frac{\partial F_{i0}}{\partial X} - \frac{e}{m_i} \frac{\sqrt{\eta_i}\omega_{ci}}{v_\perp} \frac{\partial \varphi}{\partial \phi_1} \frac{\partial F_{i0}}{\partial v_\perp} + \frac{e}{m_i} \frac{\partial \varphi}{\partial z_1} \frac{\partial F_{i0}}{\partial v_z}\right)^{-1} df_i .
\end{aligned}
$$

The potential φ has a form,

$$
\begin{aligned}
\varphi\left(\mathbf{r}, t\right) &= \int d\mathbf{k} \varphi\left(\mathbf{k}, t\right) e^{ik_x x + ik_y y + ik_z z} \\
&= \int d\mathbf{k} \varphi\left(\mathbf{k}, t\right) \exp\left[i\Omega - i\frac{k_\perp\left(t\right) v_\perp}{\omega_{c\alpha}} \sin\left(\bar{\phi} - \omega_{c\alpha} t - \theta\left(t\right)\right)\right].
\end{aligned}
$$

$$
\Omega = k_x \bar{X} + k_y \bar{Y} + k_z z - in\left(\phi_1 - \sqrt{\eta}\omega_{ci} t_1 - \theta\left(t_1\right)\right) + \mathbf{k}\left(t\right) \delta\mathbf{r}\left(t\right)
$$

and $\mathbf{k}\left(t\right) \delta\mathbf{r}\left(t\right)$ denotes the phase shift resulted from the distortion of the waves pattern by the shearing flow and the perturbations of ion trajectories due to their interaction with the ensemble of the shearing perturbations.

At times $t > \left(V_0'\right)^{-1}$ the non-modal effects determine the nonlinear evolution of drift turbulence with dominant breakdown of phase of the potential due to scattering of the angle of the gyromotion in velocity space.

$$
\begin{aligned}
f_\alpha\left(t, k_x, k_y, k_z, v_\perp, \phi, v_z, z_1\right) &= i\frac{e_\alpha}{m_\alpha}\int^{t} dt_1 \varphi\left(t_1, k_x, k_y, k_z\right) \\
&\times \exp\Big(- ik_z v_z\left(t - t_1\right) - \frac{1}{2}\left\langle\left(\mathbf{k}\left(t\right)\delta\mathbf{r}\left(t\right) - \mathbf{k}\left(t_1\right)\delta\mathbf{r}\left(t_1\right)\right)^2\right\rangle\Big) \\
&\times J_0\left(\frac{k_\perp\left(t\right)v_\perp}{\omega_c}\right) J_0\left(\frac{k_\perp\left(t_1\right)v_\perp}{\omega_c}\right)\left[\frac{k_y}{\omega_{c\alpha}}\frac{\partial F_{0\alpha}}{\partial X_\alpha} + k_{1z}\frac{\partial F_{0\alpha}}{\partial v_z}\right] \\
&+ f_\alpha\left(t = t_0, k_x, k_y, k_z, v_\perp \phi, v_z\right).
\end{aligned}
$$

At times $t > \left(V_0'\right)^{-1}$ for $k_\perp(t)\, v_\perp < \omega_{ci}$

$$\left| \frac{k_x \delta X(t)}{k_\perp(t)\, \dfrac{v_\perp}{\omega_c} \delta\phi(t)} \right| \sim \frac{1}{k_y \rho_i} \frac{1}{\left(V_0' t\right)^3}$$

and for $k_\perp(t)\, v_\perp > \omega_{ci}$,

$$\left| \frac{k_x \delta X(t)}{k_\perp(t)\, \dfrac{v_\perp}{\omega_c} \delta\phi(t)} \right| \sim \frac{1}{\left(V_0' t\right)^2}.$$

At times $t > \left(V_0'\right)^{-1}$ turbulent scattering of the angle $\delta\phi(t)$ is the dominant process in the formation the turbulent shift of the phase of the electrostatic potential.

$$\varphi(\mathbf{k},t) = \varphi_0 \exp\left[-i\omega(\mathbf{k})\,t\left(1 - \frac{(1+\tau)}{a_i b_i}\frac{t^2}{3t_s^2}\Theta(t)\right)\right.$$

$$\left. +\gamma(\mathbf{k})\,t - \frac{t^2\Theta(t)}{2a_i t_s^2} - \int\limits_0^t C(\mathbf{k},t_1)\,dt_1\right].$$

$$C(\mathbf{k},t) = \frac{c^2}{B^2}k_y^2\rho_i^2\frac{(V_0' t)^6}{8}\int d\mathbf{k}_1\,|\varphi(\mathbf{k}_1,t)|^2\,C(\mathbf{k}_1,t)\,\frac{k_{1y}^4}{\omega^2(\mathbf{k}_1)}$$

The potential ceases to grow, $\partial\varphi/\partial t = 0$, when $\gamma(\mathbf{k}) = C(\mathbf{k},t)$,

$$\frac{\gamma(\mathbf{k})}{(V_0' t)^6} = \frac{c^2}{8B^2}k_y^2\rho_i^2\int d\mathbf{k}_1\,|\varphi(\mathbf{k}_1,t)|^2\,\gamma(\mathbf{k}_1)\,\frac{k_{1y}^4}{\omega^2(\mathbf{k}_1)}$$

The non–modal evolution of the kinetic ion temperature gradient instability of the shear flow across the magnetic field

$$\int_{t_0}^{t} dt_1 \left\{ (1+T)\frac{\partial \varphi(\mathbf{k},t_1)}{\partial t_1} - \frac{\partial}{\partial t_1}\left[\varphi(\mathbf{k},t_1) A_{0i}(t,t_1)\left(1+\frac{i}{2}k_y v_{di}\eta_i(t-t_1)\right)\right] e^{-\frac{1}{2}k_z^2 v_{Ti}^2 (t-t_1)^2} \right\}$$

$$= i\int_{t_0}^{t} dt_1 \varphi(\mathbf{k},t_1) e^{-\frac{1}{2}k_z^2 v_{Ti}^2 (t-t_1)^2} \left[k_y v_{di} A_{0i}(t,t_1) - \eta_i A_{1i}(t,t_1)\right]$$

$$+T\int_{t_0}^{t} dt_1 \left[\frac{\partial \varphi(\mathbf{k},t_1)}{\partial t_1} + i k_y v_{de}\varphi(\mathbf{k},t_1)\right] e^{-\frac{1}{2}k_z^2 v_{Te}^2 (t-t_1)^2} + \varphi(\mathbf{k},t_0)\, Q(t,t_0)\,.$$

$$Q(t,t_0) = A_{0i}(t,t_0)\, e^{-\frac{1}{2}k_z^2 v_{Ti}^2 (t-t_0)^2}\left(1+\frac{i}{2}k_y v_{di}\eta_i(t-t_0)\right) - A_{0i}(t_0,t_0)\,,$$

$$A_{0i}(t,t_1) = I_0\left(\hat{k}_\perp(t)\,\hat{k}_\perp(t_1)\,\rho_i^2\right) e^{-\frac{1}{2}\rho_i^2\left(\hat{k}_\perp^2(t)+\hat{k}_\perp^2(t_1)\right)}$$

$$A_{1i}(t,t_1) = e^{-\frac{1}{2}\rho_i^2\left(\hat{k}_\perp^2(t)+\hat{k}_\perp^2(t_1)\right)}$$

$$\times\left[\frac{1}{2}\rho_i^2\left(\hat{k}_\perp^2(t)+\hat{k}_\perp^2(t_1)\right) I_0\left(\hat{k}_\perp(t)\,\hat{k}_\perp(t_1)\,\rho_i^2\right) - \rho_i^2 k_\perp(t)\,\hat{k}_\perp(t_1)\, I_1\left(\hat{k}_\perp(t)\,\hat{k}_\perp(t_1)\,\rho_i^2\right)\right]$$

where I_0 and I_1 are the modified Bessel functions of the first kind and orders 0 and 1, respectively.

$$
\begin{aligned}
\varepsilon(\mathbf{k},\omega) = 1 + T + i\sqrt{\frac{\pi}{2}}\frac{\left(\omega - k_y v_{di}\left(1-\frac{\eta_i}{2}\right)\right)}{k_z v_{Ti}} W(z_i)\, I_0\left(k_\perp^2\rho_i^2\right) e^{-\rho_i^2 k_\perp^2} \\
-z_i \frac{k_y v_{di}\eta_i}{\sqrt{2} k_z v_{Ti}}\left(1 + i\sqrt{\frac{\pi}{2}} z_i W(z_i)\right) I_0\left(k_\perp^2\rho_i^2\right) e^{-\rho_i^2 k_\perp^2} \\
+i\sqrt{\frac{\pi}{2}}\frac{k_y v_{di}\eta_i}{k_z v_{Ti}} W(z_i)\, k_\perp^2\rho_i^2 e^{-k_\perp^2\rho_i^2}\left(I_0\left(k_\perp^2\rho_i^2\right) - I_1\left(k_\perp^2\rho_i^2\right)\right) \\
+iT\sqrt{\frac{\pi}{2}}\frac{(\omega - k_y v_{de})}{k_z v_{Te}} W(z_e) = 0,
\end{aligned}
$$

The maximum growth rate $\gamma(\mathbf{k})$ of this instability is of the order of the frequency; it attains for the perturbations, which have the phase velocity along the magnetic field of the order of the ion thermal velocity, i.e.

$$
\omega(\mathbf{k}) \sim \gamma(\mathbf{k}) \sim k_z v_{Ti},
$$

and the normalized wave number component across the magnetic field, $k_\perp \rho_i$, comparable with the unity.

$$
\frac{e\left|\widetilde{\varphi}\right|}{T_i} \sim \frac{k_{0z}}{k_{0\perp}} \sim \frac{\omega(\mathbf{k}_0)}{\omega_{ci}},
$$

where the estimates $k_{0z} \sim \omega(\mathbf{k}_0)/v_{Ti}$ and $k_{0\perp} \sim \rho_i^{-1}$ were used.

For time $t > t_0 > t_s$,

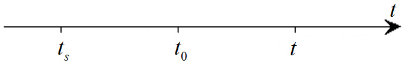

Figure 2: The domain of the integration $[t_0, t]$ over time for the short wavelength perturbations with $\hat{k}_\perp(t_0)\,\rho_i \gg 1$.

$$\varphi(\mathbf{k}, t) = \varphi_0(\mathbf{k}) \exp\left[\frac{1}{(1+T)}\frac{t_s}{t}\left(i\frac{k_y v_{di}\left(1-\frac{1}{2}\eta_i\right)}{2\kappa_i} + \frac{1}{\sqrt{2\pi}t_s^2\kappa_i^2} - \frac{k_z^2 v_{Ti}^2}{\kappa_i^3}\frac{1}{4t}\right)\right].$$

Obtained solution is very different from a canonical modal form.The non-modal effect of the time dependent ion Larmor radius gradually transforms the initially unstable modal solution $\backsim \exp\left(-i\omega(\mathbf{k})t + \gamma(\mathbf{k})t\right)$ to the non-modal zero–frequency cell–like perturbation. In the experimental condition, that evolution observes as a vanishing of the frequency spectrum of the unstable electrostatic perturbations during the time of the order of a few waves periods.

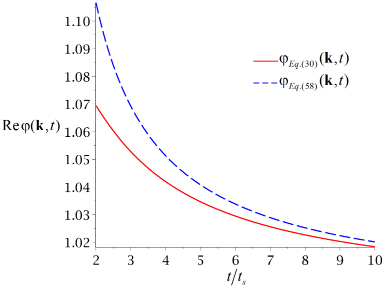

Figure 3: Linear (red) and renormalized nonlinear (blue) solutions for the potential $\varphi_{Eq.(30)}(\mathbf{k}, t)$

The nonlinear non–modal evolution of the ion temperature gradient driven turbulence in the shear flow.

$$
\begin{aligned}
&\int_{t_0}^{t} dt_1 \left\{ (1+T)\frac{\partial \varphi(\mathbf{k},t_1)}{\partial t_1} - \frac{\partial}{\partial t_1}\left[\varphi(\mathbf{k},t_1) A_{0i}(t,t_1) \right.\right.\\
&\left.\left.\times e^{-\frac{1}{2}\left\langle (\mathbf{k}(t)\delta\mathbf{r}(t)-\mathbf{k}(t_1)\delta\mathbf{r}(t_1))^2\right\rangle}\left(1+\frac{i}{2}k_y v_{di}\eta_i(t-t_1)\right)\right] e^{-\frac{1}{2}k_z^2 v_{Ti}^2 (t-t_1)^2}\right\}\\
&= i\int_{t_0}^{t} dt_1 \varphi(\mathbf{k},t_1) e^{-\frac{1}{2}k_z^2 v_{Ti}^2 (t-t_1)^2} e^{-\frac{1}{2}\left\langle (\mathbf{k}(t)\delta\mathbf{r}(t)-\mathbf{k}(t_1)\delta\mathbf{r}(t_1))^2\right\rangle} k_y v_{di}\\
&\times\left[(1-\eta_i) A_{0i}(t,t_1) + \eta_i A_{1i}(t,t_1)\right]\\
&+T\int_{t_0}^{t} dt_1 \left[\frac{\partial \varphi(\mathbf{k},t_1)}{\partial t_1} + i k_y v_{de}\varphi(\mathbf{k},t_1)\right] e^{-\frac{1}{2}k_z^2 v_{Te}^2 (t-t_1)^2}\\
&+\varphi(\mathbf{k},t_0) Q(t,t_0).
\end{aligned}
$$

$$\left\langle \left(\mathbf{k}(t)\,\delta\mathbf{r}(t)-\mathbf{k}(t_1)\,\delta\mathbf{r}(t_1)\right)^2\right\rangle = \left\langle \left(\mathbf{k}(t)\,\delta\mathbf{r}(t)-\mathbf{k}(t_1)\,\delta\mathbf{r}(t_1)\right)^2\right\rangle_0$$
$$+\frac{v_\perp^2}{2\omega_{ci}^2}\left\langle \left(\hat{k}_\perp(t)\,\delta\phi(t)-\hat{k}_\perp(t_1)\,\delta\phi(t_1)\right)^2\right\rangle .$$

At time $t > \left(V_0'\right)^{-1}$, the nonlinear scattering of the Larmor gyrophase $\delta\phi(t)$ becomes the dominant process in the formation of the nonlinear shift of the phase of the electrostatic potential.

$$\left\langle \left(\mathbf{k}(t)\,\delta\mathbf{r}(t)-\mathbf{k}(t_1)\,\delta\mathbf{r}(t_1)\right)^2\right\rangle \approx \frac{v_\perp^2}{2\omega_{ci}^2}\left\langle \left(\hat{k}_\perp(t)\left(\delta\phi(t)-\delta\phi(t_1)\right)\right)^2\right\rangle$$
$$\sim \left(\omega(\mathbf{k}_0)\,t\right)^2\frac{\left(t^{3/2}-t_1^{3/2}\right)^2}{t_s^3}.$$

$$\varphi(\mathbf{k},t)=\varphi_0(\mathbf{k})\exp\left[\frac{1}{(1+T)}\left(\frac{ik_y v_{di}}{3\sqrt{2}\omega(\mathbf{k}_0)}\left(\frac{t_s}{t}\right)^{5/2}\right.\right.$$
$$\left.\left.+\frac{t_s}{\sqrt{2\pi}t}+\frac{2t_s^2}{9\sqrt{2\pi}\omega^2(\mathbf{k}_0)\,t^4}\right)\right].$$

The suppression of the anomalous transport by a shear flow.

$$\frac{\partial F_{i0}}{\partial t} \approx \frac{e^2}{m_i^2} \int d\mathbf{k} \left|\varphi_0\left(\mathbf{k}\right)\right|^2 \frac{1}{6\sqrt{2\pi}\omega\left(\mathbf{k}_0\right)} \left(\frac{t_s}{t}\right)^{5/2} \frac{v_{Ti}}{v_\perp}$$
$$\times \left(\frac{k_y}{\omega_{ci}}\frac{\partial}{\partial \bar{X}} + k_z \frac{\partial}{\partial v_z}\right)\left(\frac{k_y}{\omega_{ci}}\frac{\partial F_{i0}}{\partial \bar{X}} + k_z \frac{\partial F_{i0}}{\partial v_z}\right).$$

$$n_{0i}\frac{\partial T_i}{\partial t} \approx K_i\left(t\right)\frac{\partial^2 T_i}{\partial \bar{X}^2}.$$

The order of value estimate for the anomalous ions thermal conductivity $K_i\left(t\right)$,

$$K_i\left(t\right) = \frac{e^2}{T_i^2}\int d\mathbf{k}\left|\varphi_0\left(\mathbf{k}\right)\right|^2 \frac{v_{Ti}^2 k_y^2 \rho_i^2}{6\sqrt{2\pi}\omega\left(\mathbf{k}_0\right)}\left(\frac{t_s}{t}\right)^{5/2},$$

which displays the decay with time, that is a strictly non-modal effect, originated from the sheared modes structure of the shear flow turbulence.

Conclusions

1. The application of the approximation of the "slow" spatial variation of the flow velocity is valid only for time $t \ll (V_0')^{-1}$. The "quench rule" predicts the suppression of the turbulence when $V_0' > \gamma$. For time $t \sim \gamma^{-1}$ it occurs when $V_0' t > 1$, i.e. under conditions at which modal approach is not valid. The suppression of the turbulence by the shear flow is the non-modal process.

2. At these conditions the solution to Vlasov equation can't be presented in the laboratory coordinates in a form, in which the time and spatial dependences are separable.

3. The solution to Vlasov equation in the form of the separate Fourier harmonic with time independent wave numbers may be obtained only in convected-sheared coordinates. That solution reveals in the laboratory frame as a shearing mode with time dependent x-component of the wave number.

4. The time dependence of the finite Larmor radius effect is the basic linear mechanism of the action of the velocity shear on waves and instabilities in plasma shear flow. The drift kinetic equation does not contain any effects conditioned by shear flow.

5. The dominant process which is responsible for the rapid suppression of the kinetic instabilities is the turbulent scattering of the angle of the ion gyromotion by the ensemble of the shearing waves with random phases.